# Nonequilibrium properties of ultrashort superconducting microbridges

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Superconducting microbridges with lengths ranging from shorter than to longer than the coherence length have been fabricated. Those microbridges whose length is longer than the coherence length do show a dynamic enhanced critical current in accordance with the theory of Schmid, Schön, and Tinkham. Those microbridges whose length is shorter than the coherence length do not exhibit a dynamically enhanced critical current and a possible reason for its absence is that the order parameter is depressed in the banks adjacent to the microbridge.

## INTRODUCTION

Superconducting contacts (microbridges, weak links, point contacts, etc.) exhibit a great variety of physical phenomena which have been the object of much study. Pair coupling exists across these contacts in a similar way to that which occurs across Josephson thin-film junctions, but with the exception that a large quasiparticle current exists through the contact.

The resistively shunted junction (RSJ) model[1] was introduced to explain the equilibrium properties of the superconducting contact, and when discrepancies between theory and experiment were noted it was postulated that there were nonequilibrium states of the quasiparticles and pairs occurring within the contacts. Subsequent theoretical models of Aslamazov and Larkin,[2] Golub,[3] and Schmid *et al.*[4] were introduced, and these take into account the nonequilibrium properties of the contact. The starting point for all of these theories is time-dependent Ginzburg-Landau theory. In this paper we describe a new technique for preparing variable thickness microbridges that are as short as 0.02 $\mu$m and whose length is well determined. Current-voltage data from these junctions is presented. Reasonable agreement with the nonequilibrium theories exist except for the shortest microbridges. Qualitative reasons for this discrepancy are discussed.

## EXPERIMENT

In the first step of the preparation of a microbridge, a tin layer (1000—5000 Å) was deposited by thin-film evaporation onto a glass slide (Fig. 1). This formed one bank of the microbridge. A layer of silicon monoxide (200—10 000 Å) was evaporated over a portion of the first layer and the thickness of this layer determined the length of the microbridge. An electrolytically etched point[5] on a tungsten wire was used to make a hole through the silicon-monoxide layer until contact was made with the lower tin layer. A second tin layer was evaporated over the silicon-monoxide layer and the hole was filled during this evaporation. The filled hole formed the microbridge and the second tin layer became the upper bank of the microbridge. With this technique, microbridges of about 1-$\mu$m diameter and lengths 0.02 to 1 $\mu$m could be fabricated. These variable thickness microbridges[6] have banks that are much thicker than the bridge length and they have the advantage that they are able to sustain higher levels of heat dissipation than planar microbridges. A similar fabrication technique has been described by Gamo *et al.*,[7] who made the hole through the insulating layer using an ion beam.

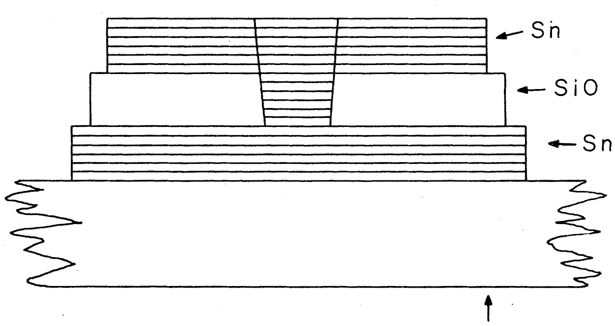


FIG. 1. Schematic view of the microbridge.

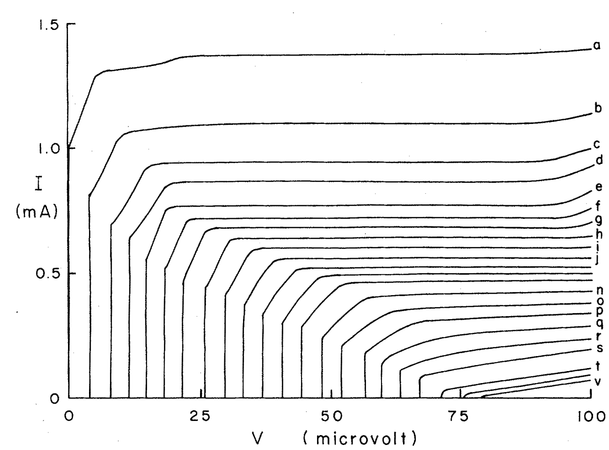


FIG. 2. Current-voltage characteristics measured on a microbridge 0.6 $\mu$m long. The curves are displaced sideways from each other by 3.75 $\mu$V for clarity at the following temperatures: (a) 3.289, (b) 3.387, (c) 3.438, (d) 3.464, (e) 3.511, (f) 3.527, (g) 3.542, (h) 3.558, (i) 3.575, (j) 3.592, (k) 3.607, (l) 3.618, (m) 3.630, (n) 3.649, (o) 3.666, (p) 3.679, (q) 3.694, (r) 3.710, (s) 3.722, (t) 3.743, (u) 3.750, (v) 3.760.

 

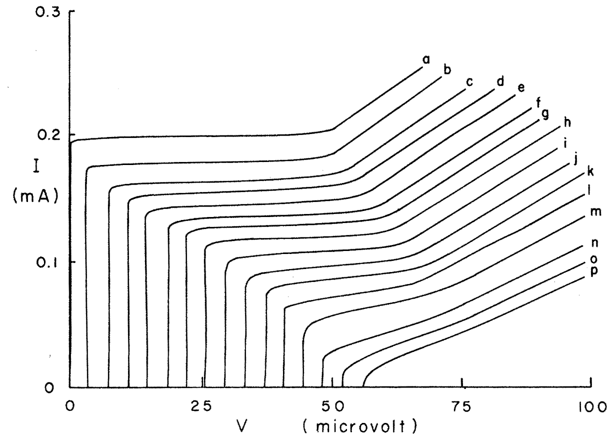


FIG. 3. Current-voltage characteristics measured on a microbridge 0.05 $\mu$m long. The curves are displaced sideways from each other by 3.75 $\mu$V for clarity at the following temperatures: (a) 3.503, (b) 3.535, (c) 3.556, (d) 3.569, (e) 3.584, (f) 3.597, (g) 3.607, (h) 3.621, (i) 3.638, (j) 3.655, (k) 3.669, (l) 3.689, (m) 3.704, (n) 3.737, (o) 3.751, (p) 3.759.

Figures 2 and 3 are the current-voltage characteristics of two tin microbridges of lengths 0.05 and 0.6 $\mu$m. For both microbridges the usual subharmonic gap structure was observed at higher potentials. An interesting feature of these two sets of current-voltage characteristics is the presence of a dynamically enhanced critical-current structure[8] at low potentials in the long microbridge (Fig. 2) and its absence in the short microbridge (Fig. 3). (Octavio *et al.*[8] termed the dynamically enhanced critical current the "foot".)

## DISCUSSION

The RSJ model encompasses the features of the Josephson effect necessary to describe the equilibrium conditions in a microbridge. When the critical current is exceeded, an ac potential develops across the microbridge with a frequency 2 eV/h$=\tau_j^{-1}$. The density of pairs and quasiparticles in the microbridge will attempt to fluctuate at the same frequency. If the density of pairs and quasiparticles are unable to fluctuate at the Josephson frequency, then a nonequilibrium condition will occur.

The various nonequilibrium theories[2–4] are extensions of time-dependent Ginzburg-Landau (TDGL) theory. Aslamazov and Larkin[2] gave a description that included a nonequilibrium electron distribution function together with the BCS self-consistent gap equation. As they considered time-averaged quantities their predictions are restricted to high voltages. Golub's[3] approach, also based on TDGL, uses diffusion as the main mechanism of dissipation of the nonequilibrium states in the bridge but does not include inelastic scattering. The work of Schmid, Schön, and Tinkham[4] follows on similar lines to that of Golub but they have included inelastic scattering. These two latter theories should apply at low voltages. Extensive experimental work was carried by Octavio *et al.*[8] using microbridges whose lengths were as small as 0.5 $\mu$m. They compared experimental data on the dynamically enhanced critical-current structure, which occurs at low voltages in the current-voltage characteristic, to predictions of Schmid, Schön, and Tinkham, and they found considerable agreement.

In the model of Schmid, Schön, and Tinkham[4], diffusion and inelastic scattering are considered to be the most important processes in relaxing the nonequilibrium pair and quasiparticle populations. The starting point of this model is to consider a superconductor with a short mean free path $\tau_{\text{imp}}T_c \ll 1$ ($\tau_{\text{imp}}$ is the impurity scattering time; $T_c$ is the critical temperature of the bulk) where the local state of the microbridge depends only on the distance from the banks.

The order parameter in the banks of a microbridge is assumed to be the bulk equilibrium value $\Delta_b$. While in the center of the microbridge the order parameter is depressed because of the high current density. The low-energy quasiparticle excitations (close to the energy-gap edge) play an important role in the enhancement of the gap. Because of the low value of the energy gap within the center of a microbridge those quasiparticles with energy less than $\Delta_b$ are trapped within the microbridge. The two possible mechanisms that could enable them to relax are inelastic electron-phonon scattering and diffusion. Diffusion of a quasiparticle with energy less than $\Delta_b$ from within the microbridge to the banks is not possible because there are not any states of the same energy in the banks. Diffusion can only take place between states of the same energy. Thus relaxation of quasiparticles of energy less than $\Delta_b$ from within the microbridge can only occur by inelastic electron-phonon scattering. In a medium-sized microbridge with a small potential ($<1$ $\mu$V) across it, the Josephson period $\tau_j$ is longer than the inelastic scattering time $\tau_E$. Because there is a potential across the microbridge the density of pairs and quasiparticles will attempt to oscillate at the Josephson frequency. The interchange from pair to quasiparticle and vice versa must occur by inelastic electron-phonon scattering. This process will occur at low potentials when $\tau_E < \tau_J$. In this case nonequilibrium effects do not occur. However, at higher potentials $\tau_J$ is shorter and $\tau_E > \tau_J$. The interchange of pairs and quasiparticles is not able to occur and nonequilibrium exists. The excess of nonequilibrium quasiparticles gives rise to an increased potential across the microbridge. This is the dynamically enhanced critical current of the *I-V* characteristic. To describe this dynamic process, Schmid, Schön, and Tinkham used two Boltzmann equations in a way similar to the method introduced by Larkin and Ovchnnikov[9] and Schmid and Schön,[10] who split the distribution function in two components: the longitudinal one is associated with the variation of the modulus of the order parameter, while the transverse one is related to the gauge invariant part, or in other words, to the phase difference of the order parameter. The supercurrent is determined by the quasiparticles at the gap edge. The correction to the supercurrent for finite length was also calculated. The ratio of the enhanced supercurrent to the critical current is given in this model by Eq. (1):

$$\frac{I_{c1}}{I_{c0}} = 1.325 + 0.019\frac{\Delta_b}{kT}\left[\frac{L}{\xi(0)}\right]^2 , \tag{1}$$

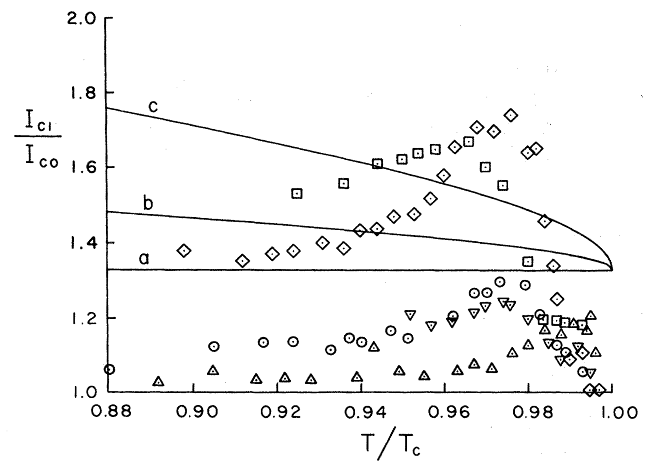


FIG. 4. $I_{c1}/I_{c0}$ versus $T/T_c$. The continuous lines are calculated from Eq. (1) for lengths $a$, 0.05 $\mu$m; $b$, 0.60 $\mu$m; and $c$, 1.0 $\mu$m. The points are experimental data for microbridges of length 0.05 $\mu$m (◬), 0.1 $\mu$m (⊙), 0.2 $\mu$m (⊽), and 0.6 $\mu$m (◈,⊡).

where $L$ is the length of the microbridge, $T$ is the temperature, $\Delta_b$ is the energy gap, and $\xi(0)$ is the coherence length. $I_{c1}$ is the enhanced critical current. Data for a number of microbridges together with the ratio $I_{c1}/I_{c0}$ calculated from Eq. (1) was plotted in Fig. 4. The $I_{c1}/I_{c0}$ data for the long microbridges (0.6 $\mu$m) are all greater than 1.325 except at temperatures very close to $T_c$. The temperature variation does not fall close to the theoretical prediction, curve $c$. However, the variation shows a similarity to other experimental data of Octavio *et al.*,[8] displaying a rise close to $T/T_c \rightarrow 1$ and falling down to 1.325 as $T/T_c$ decreases. On the other hand, for short microbridges (0.05, 0.1, and 0.2 $\mu$m) the data falls below the $I_{c1}/I_{c0}=1.325$ line as would be expected as the dynamically enhanced critical current is either much reduced or absent for these microbridges. The normalized

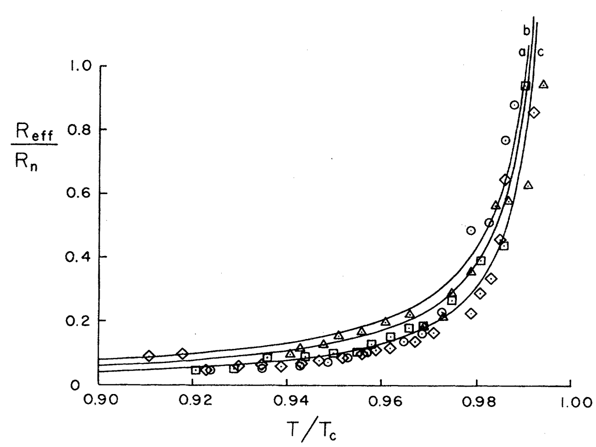


FIG. 5. $R_{\rm eff}/R_n$ versus $T/T_c$. The continuous lines are calculated from Eq. (2) for lengths $a$, 0.2 $\mu$m; $b$, 0.6 $\mu$m; $c$, 1.0 $\mu$m. The points are experimental data for microbridges of length 0.2 $\mu$m (◬,⊡) and 0.6 $\mu$m (⊙,◈).

differential resistance of the dynamically enhanced-critical-current region is given by

$$\frac{R_{\rm eff}}{R_N} = 10.8 \frac{\hbar}{kT_c \tau_E} \left[ \frac{kT_c}{\Delta_b} \right]^2 \times \left[ 1 + 0.052 \frac{\Delta_b}{kT_c} \left[ \frac{L}{\xi(0)} \right]^2 \right]^{-1} , \quad (2)$$

where $\tau_E$ is the electron-phonon collision time. The ratio $R_{\rm eff}/R_N$ calculated from Eq. (2) together with experimental data is plotted in Fig. 5. Those microbridges whose length is shorter than 0.1 $\mu$m do not display a dynamically enhanced critical current and consequently it is not possible to estimate a value of $R_{\rm eff}$ for these short microbridges. The agreement between experimental and theoretical data is quite good, but keeping in mind that only microbridges whose length is greater or comparable to the coherence length are included.

The effect of thermal fluctuations on the dc Josephson effect in a junction or microbridge of small capacitance is to cause rounding of the $I$-$V$ characteristic near zero voltage. Ambegaokar and Halperin[11] have calculated this rounding, and following their description we found that a noise temperature of about 60 K accounts for the rounding of those $I$-$V$ curves taken at 3.7 K. For those $I$-$V$ characteristics taken at lower temperatures the rounding due to thermal fluctuations is negligible.

In the work of Schmid, Schön, and Tinkham,[4] the energy gap in the banks of the microbridge is assumed to be $\Delta_b$, the bulk value, irrespective of the length of the microbridge. Blackburn *et al.*[12] calculated the order parameter throughout a microbridge and its banks using Ginzburg-Landau theory and Zaitsev[13] boundary conditions. They concluded that for microbridges that are shorter than the coherence length the order parameter is unable to vary significantly within the microbridge and much of the variation of the order parameter occurs outside of the microbridge. In other words, the order parameter in the banks is much less than the bulk equilibrium value. Consequently, the gap parameter in the banks adjacent to the microbridge will be much less than $\Delta_b$. Conversely for microbridges whose length is greater than the coherence length, the gap parameter in the banks adjacent to the microbridge will approach $\Delta_b$. The coherence length of tin is approximately 0.23 $\mu$m. We would expect those microbridges of length (0.6 $\mu$m) greater than the coherence to follow the Schmid, Schön, and Tinkham theory. Those microbridges of length shorter than the coherence will have significant deviation of the energy gap in the banks from the bulk value with the result that the gap value in the microbridge is only slightly depressed from the value of the gap in the region of the banks adjacent to the microbridge. The dynamically enhanced critical current will be much reduced for these microbridges as the data reveals.

In conclusion, we have presented data showing the presence of a dynamically enhanced critical current in the current-voltage characteristics of microbridges longer than 0.6 μm and the absence of the dynamically enhanced critical current in the current-voltage characteristic of microbridges as short as 0.05 μm. We suggest that the absence of a dynamically enhanced critical current in the short microbridges may be due to the depression of the gap in the region of the banks adjacent to the microbridge.

## ACKNOWLEDGMENTS

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