Relaxation Phenomena in Viscoelastic Colloidal Suspensions with Internal Rotation

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In this paper, extended irreversible thermodynamics is used to obtain the constitutive equations which govern the motion and relaxation processes of a suspension of particles with internal rotation in a viscoelastic fluid. The system of constitutive equations may be particularized to obtain several cases mentioned before in the literature as those given by Shliomis, Eringen, and Brenner. A qualitative discussion about macroscopic mass polarization diffusion is presented. The linearized constitutive equations, together with the linear momentum and angular momentum equations, are the stand point to calculate the complex viscosity in the case where the velocity is not coupled with the mass polarization. It is shown that for some values of the parameters the subsystems of the fluid and the collection of particles with spin may act independently, but for other values they can have a mutual interchange of energy. Moreover, to present a flow solution where the coupling dynamics appears explicitly, an example is given for the flow of a nonviscoelastic fluid suspension down an inclined plane. It is shown that the velocity depends linearly on the mass polarization through which the couplings control the form and magnitude of the velocity profile.

Key Words: suspensions; internal rotation; nonequilibrium thermodynamics; viscoelasticity; constitutive equations.

I. INTRODUCTION

Micropolar suspensions are composed by small particles suspended in a fluid. The particles possess an embedded permanent mass dipole. The action of the gravity field on the particle produces a couple which enhances the rotational motion of the particles. The micropolar suspension dynamics have been treated previously by Shliomis (1), Eringen (2–4), and Brenner and Sellers (5–7). They considered microfluids in the context of fluid mechanics and linear irreversible thermodynamics. In particular, Eringen has a generalization of the description of suspension with microstructured particles.

In a series of papers, in the decade of the seventies, Hynes et al. (8–10) and later Sparling and Reichl (11–12) investigated the case of Brownian particles in viscoelastic fluids with internal rotation. In those papers two levels of description are apparent. One is the microscopic regime in which generalized hydrodynamics is used. This regime has as characteristic that the colloidal particles have the same radius of the fluid particles. The diffusion of the spin is taken into account and the mass-dipolar effect can be ignored. The other level of description is the hydrodynamic limit in which the colloidal particle is larger than the fluid particle. In this limit the spin-diffusivity coefficient is considered negligible (13). The thermodynamical description of this regime was presented in the book by de Groot and Mazur (14). Our point of view in this paper is a description in which an intermediate level may be approached. We assume that there is no spin diffusion effect and that the colloidal particles are large enough to have the mass polarization diffusion effect.

Our main purpose in this kind of system is to consider relaxation and retardation phenomena in colloidal suspensions within the framework of extended irreversible thermodynamics (EIT). This theory deals with relaxation phenomena of variables with different tensorial character and mutual coupling. In our case, we investigate the interdependence between the mass polarization vector and the symmetric and antisymmetric stress tensors produced by the velocity and spin fields of a viscoelastic suspension with colloidal particles. The particle–liquid interactions as well as the diffusion of the polarization are included. From the beginning, we consider a homogeneous solution in an incompressible fluid and we do not take into account, explicitly, the interaction between particles.

The paper is structured as follows. In Section II we obtain the constitutive equations for the nonequilibrium system subject of study. Viscoelastic properties and diffusion with hydrodynamic effects are discussed as particular cases of these equations. In Section III a discussion is given for the complex viscoelastic response calculated from a combination of the linearized constitutive equations and the equations of linear and angular momentum. In Section IV, an example of a nonviscoelastic thin fluid suspension layer flowing down an inclined plane is given to show, from another point of view, the role played by the parameters appearing in the constitutive equations. In Section V we present the discus-